(Holographic) Entanglement Entropy

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DKPI Colloquim May 18^{th} 2015

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Role of EE in QFT

- Entanglement entropy as a measure of degrees of freedom.

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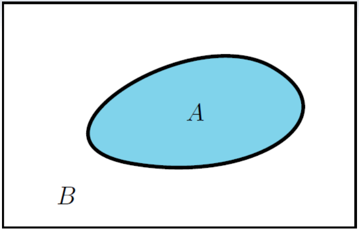
- Entanglement entropy as a measure of degrees of freedom.
- An order parameter for various phase transitions.
- Reconstruction of bulk geometry from entanglement.

Definition of EE

- Divide a System into subsystems A and $B = \bar{A}$: $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$



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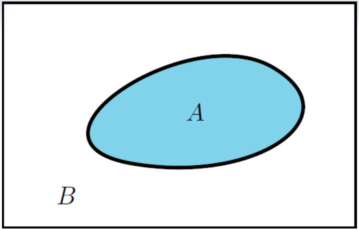

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- Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\, \rho_{tot} = \sum_i \langle\psi_B^i|\rho_{tot}|\psi_B^i\rangle$$

$$\mathcal{H}_B = \{|\psi_B^1\rangle, |\psi_B^2\rangle, \ldots\}$$

Quantum Mechanical System

Suppose $|\Psi\rangle = \sum_{i=1}^{d_A} \sum_{j=1}^{d_B} c_{ij} |\psi_A^i\rangle |\psi_B^j\rangle, \quad d_{A,B} = \dim \mathcal{H}_{A,B}$

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- For $S_A = \log \min(d_A, d_B) \quad \rightarrow$ Maximally entangled state.

Spin-2 System: Pure Product State

Consider:

$$|\Psi\rangle = \frac{1}{2}\left(|\uparrow\rangle_A + |\downarrow\rangle_A\right) \otimes \left(|\uparrow\rangle_B + |\downarrow\rangle_B\right)$$

$$\rightarrow \quad \rho_A = \mathrm{tr}_B |\Psi\rangle\langle\Psi| = \frac{1}{2}\left(|\uparrow\rangle_A + |\downarrow\rangle_A\right)\left(|\uparrow\rangle_A + |\downarrow\rangle_A\right)$$

Eigenvalues:$1, 0$ $\quad \rightarrow S_A = -\mathrm{tr} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \log(1) & 0 \\ 0 & \log(0) \end{pmatrix}$

$\rightarrow S_A = 0$

Spin-2 System: Maximally Entangled State

Consider:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right)$$

$$\rightarrow \quad \rho_A = \mathrm{tr}_B|\Psi\rangle\langle\Psi| = \frac{1}{2}\left(|\uparrow\rangle_A\langle\uparrow|_A + |\downarrow\rangle_A\langle\downarrow|_A\right)$$

Eigenvalues: $\frac{1}{2}, \frac{1}{2} \quad \rightarrow S_A = -\mathrm{tr}\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \log(\frac{1}{2}) & 0 \\ 0 & \log(\frac{1}{2}) \end{pmatrix}$

$\rightarrow S_A = \log 2$

Properties of Entanglement Entropy

Area Law in QFT$_{d+1}$

$$S_A = \frac{\text{Area}(\partial A)}{\epsilon^{d-1}} + \ldots, \quad \epsilon : \text{UV cutoff}$$

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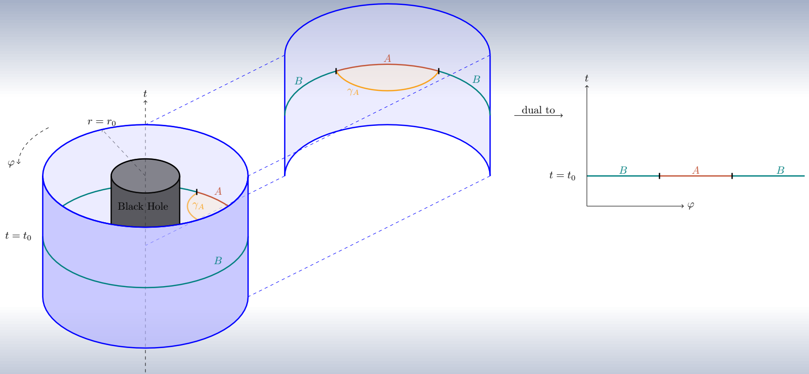
Strong Subadditivity

$$\begin{aligned} S_{A\cup B\cup C} + S_B &\leq S_{A\cup B} + S_{B\cup C} \\ S_A + S_C &\leq S_{A\cup B} + S_{B\cup C} \end{aligned}$$

for any three disjoint regions A, B and C.

Holographic Entanglement Entropy

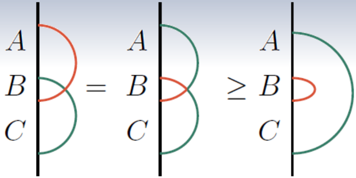


2 + 1-dimensional gravity theory with a boundary containing a 1 + 1-dimensional quantum field theory.

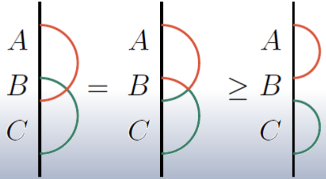
The length of the curve γA determines the entanglement entropy of A and B.

Holographic Proof of Strong Subadditivity

$$S_{A\cup B\cup C} + S_B \leq S_{A\cup B} + S_{B\cup C}$$



$$S_A + S_C \leq S_{A\cup B} + S_{B\cup C}$$



Thank You for Your Attention!