# Entanglement Entropy and AdS/CFT

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2nd DK Colloquium
January 19, 2015

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

DOKTORATSKOLLEG PI
Particles and Interactions

INSTITUTE FOR THEORETICAL PHYSICS

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# The main messages of this talk

- Entanglement entropy is a **measure for entanglement** in quantum systems.

  (Other measures have been suggested, but they are not so well suited for doing calculations.)

- Entanglement entropy is **difficult to compute in quantum field theories**.

  (Analytic results are available for 1+1 dim. CFTs.)

- The **AdS/CFT** correspondence brings a huge **simplification**:

  **Entanglement entropy = area of extremal “surfaces”**

  (This makes entanglement entropy (numerically) tractable in higher dimensional quantum field theories.)

# Outline

**1. Introduction**

- Entanglement Entropy: Definition, QM, QFT
- Holographic Principle & AdS/CFT
- Holographic Entanglement Entropy: Extremal Surfaces

**2. Applications**

- Holographic Thermalization
- Example: Holographic Quantum Revivals

**3. Summary**

# Definition of entanglement entropy

Consider a quantum system in a **pure state** $|\psi\rangle$.

The **density matrix** of this state is given by $\rho = |\psi\rangle\langle\psi|$.

**Divide** the system into **two parts** A,B.
The total Hilbert space is factorized:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

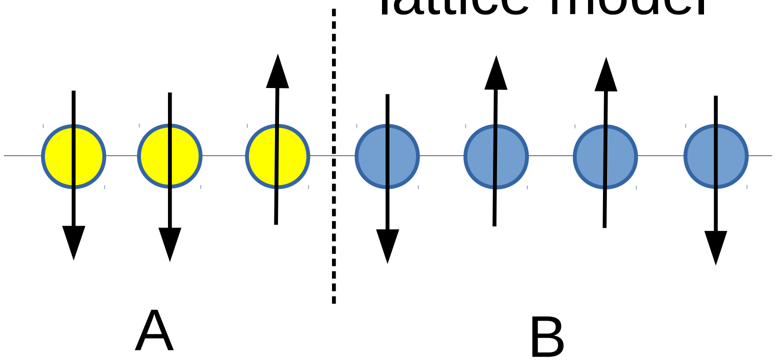
The **reduced density matrix** of A is obtained by the trace over $\mathcal{H}_\mathcal{B}$

$$\rho_A = \mathrm{Tr}_B \rho$$

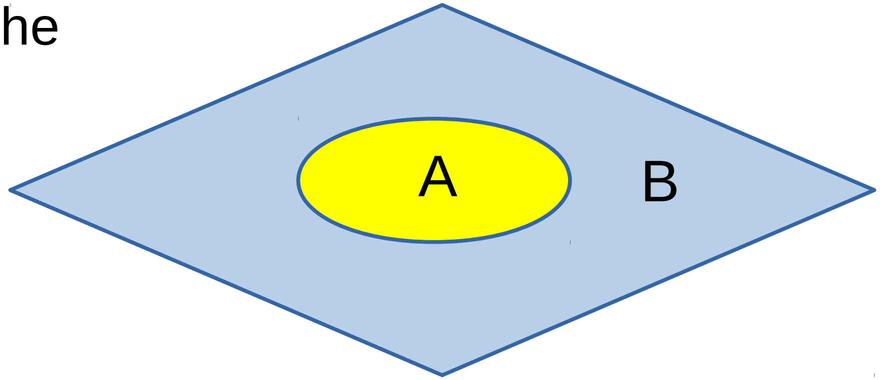
**Entanglement entropy** is defined as the **von Neumann entropy** of $\rho_A$:

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

lattice model




quantum field theory

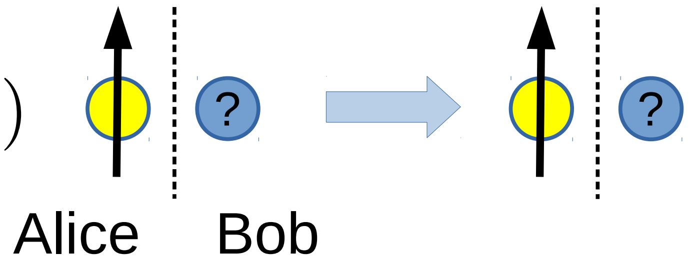


# Entanglement entropy in a two quantum bit system

Consider a quantum system of two spin 1/2 dof's.
Observer Alice has only access to one spin and Bob to the other spin.

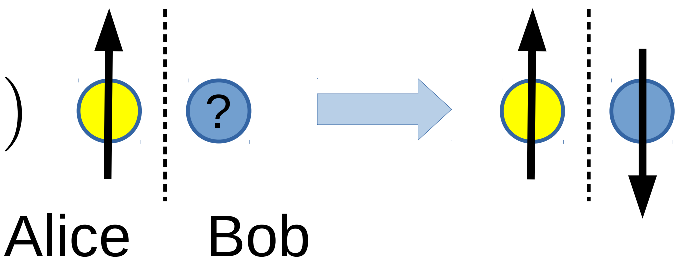
A **product state (not entangled)** in a two spin 1/2 system: $S_A = 0$

$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle + |\downarrow_A\rangle) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$




A (maximally) **entangled state** in a two spin 1/2 system: $S_A = \log 2$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle \otimes |\downarrow_B\rangle - |\downarrow_A\rangle \otimes |\uparrow_B\rangle)$$



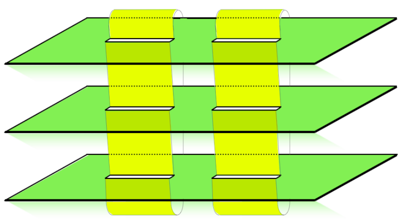

Entanglement entropy is a **measure** for how much a given quantum state is **entangled**.

# Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method.**

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated!**

With the **replica method** one gets **analytic results** for **1+1 dim. CFTs**. [Holzhey-Larsen-Wilczek 94]



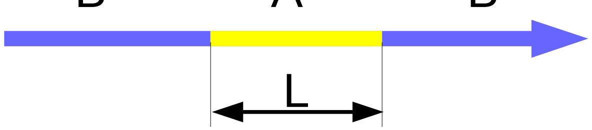
3-sheeted Riemann surface

One finds **universal scaling** with interval size:

central charge of the CFT

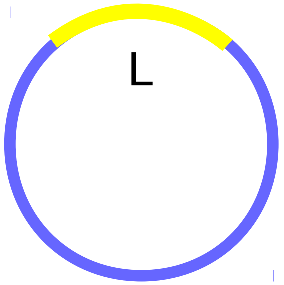
$$S_A = \frac{c}{3} \log \frac{L}{a} + finite$$

UV cut off




**Message:** Computing entanglement entropy in interacting QFTs is complicated and analytically only possible in 1+1 dim. CFTs.

1+1 dim. CFTs

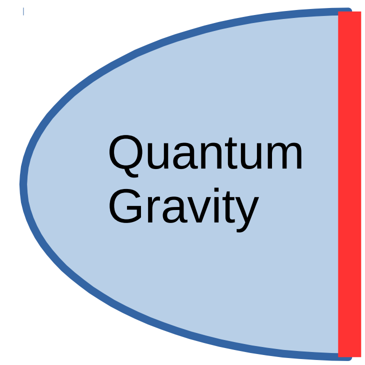



The **holographic principle** provides a **simpler method** that works also in **higher dimensions**.

# Holographic principle & AdS/CFT correspondence

**<u>Holographic principle</u>:** ['t Hooft 93, Susskind 94]

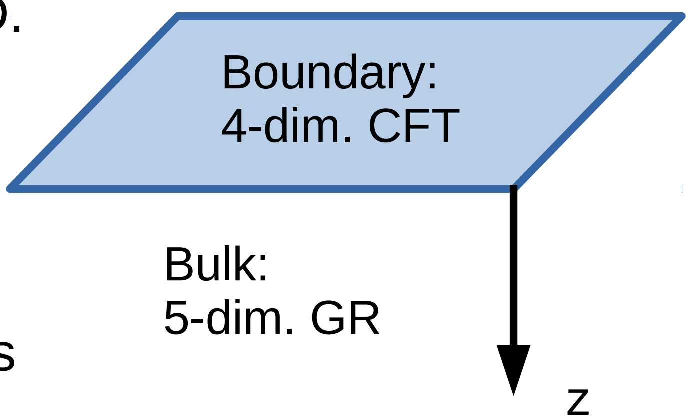
A **(d+2) dim. theory of quantum gravity** has an equivalent description in terms of a **(d+1) dim. theory without gravity**.




**<u>AdS/CFT correspondence</u>:** [Maldacena 97]

**Type IIB string theory** on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}$=4 super symmetric SU($N_c$) **Yang-Mills theory** in 4D.

The correspondence is a **strong/weak duality**.




**<u>Supergravity limit: strong coupling & large $N_c$</u>**

**Strongly coupled** large $N_c$ $\mathcal{N}$=4 SU($N_c$) SYM theory is equivalent to **classical (super)gravity** on $AdS_5$.

We can use **general relativity** to solve a **strongly coupled gauge theory**!

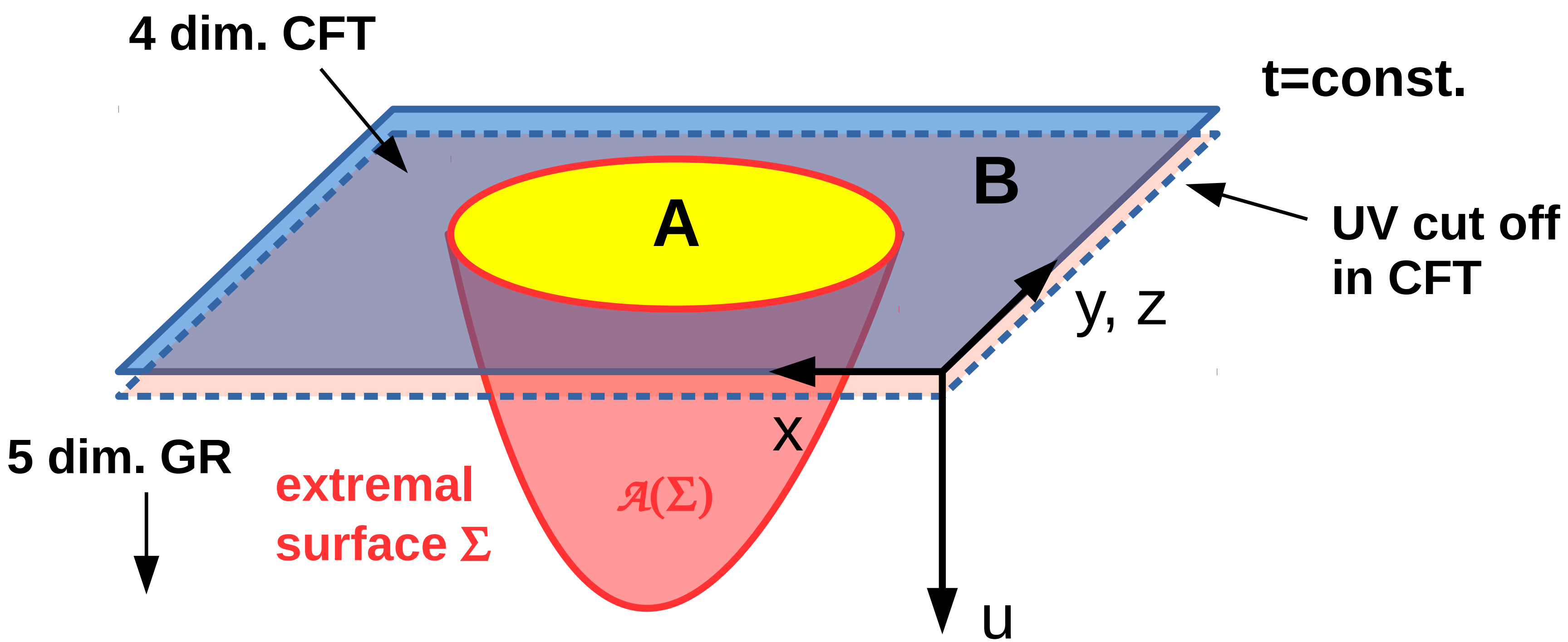
# Holographic entanglement entropy

Entanglement entropy in the **4 dim. CFT** can be computed from **extremal surfaces** in the **5 dim. gravity theory**.

(Extremal surfaces are **saddle points** of the **area functional** in the **5 dim. geometry**.)

$$S_A = \frac{\mathcal{A}(\Sigma)}{4G_N}$$

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]

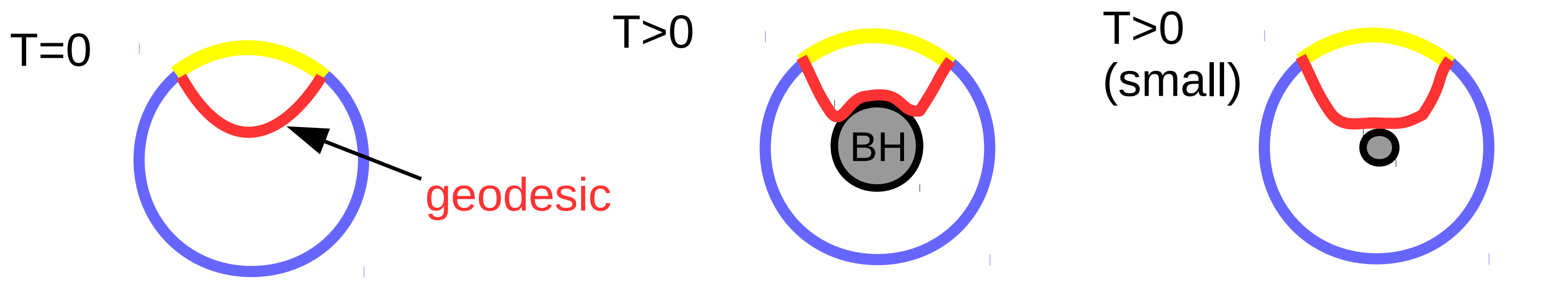



Surfaces are infinitely “long”, entanglement entropy in QFTs diverges - need a cut off.

# Some illustrative examples

<u>1+1 dim. CFT <=> 3 dim. AdS(-BH)</u>

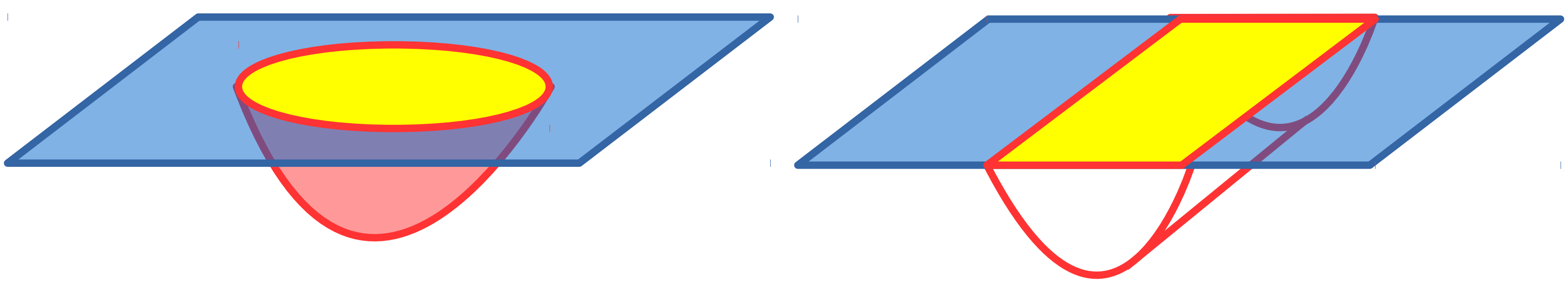
Hawking temperature of BH = T of CFT




Entanglement entropy = **length** of geodesic

<u>2+1 dim. CFT <=> 4 dim. AdS(-BH)</u>

In **higher dimensions** we can study regions with **different "shape"**



Entanglement entropy = **area** of extremal surface

<u>3+1 dim. CFT <=> 5 dim. AdS(-BH)</u>

Entanglement entropy = **"volume"** of extremal 3 dim. spatial region

# Holographic Thermalization

- The **central question**:

  How **evolves** an **excited** quantum system to **thermal equilibrium**?
- Holographic **entanglement entropy** serves as an **observable** to study **thermalization**.
- The holographic dual of **thermalization in CFT** is the **formation** of a **black hole** in the **gravity theory**.
- Has to consider **time dependent geometries** in on the gravity side → one has to do **numerical relativity**.
- To compute the time evolution of entanglement entropy one has to compute extremal surfaces in these **numerical spacetimes**.

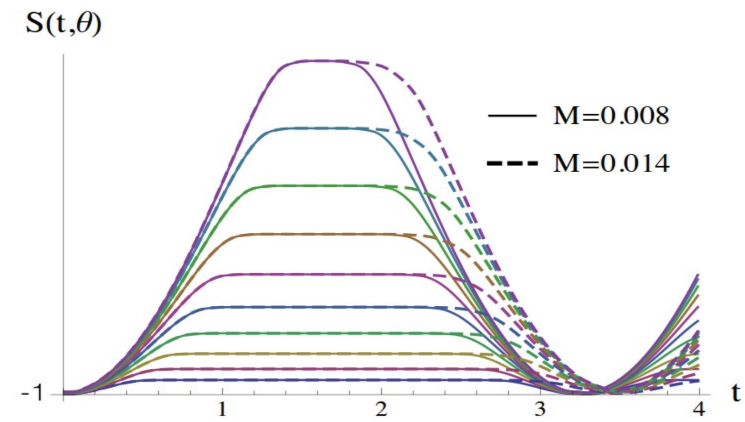
# Holographic quantum revivals

**Quantum revival**: collapse and **reappearance** of a coherent quantum state.

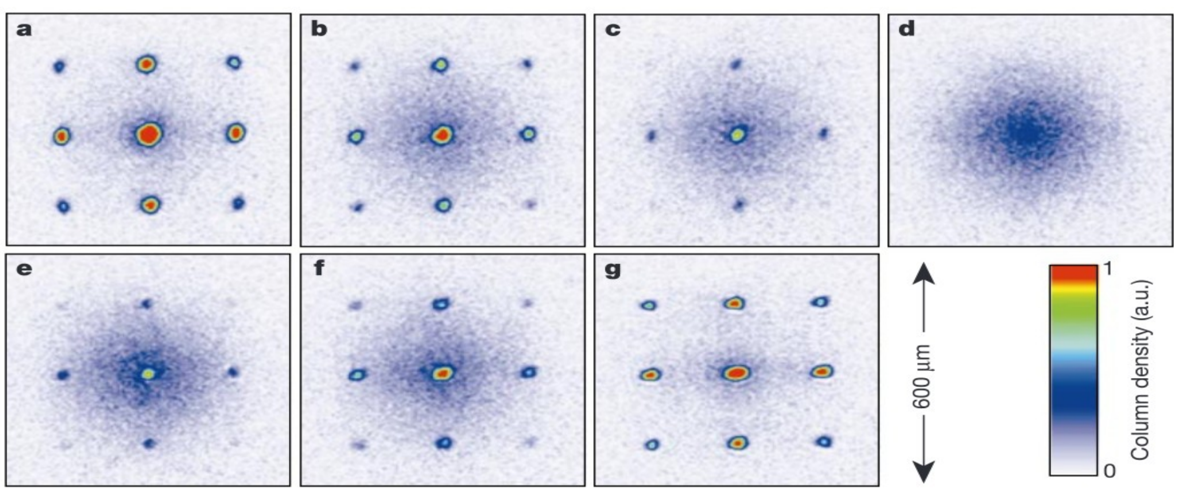
Experimentally realized in **Bose Einstein condensate** trapped in an optical lattice.

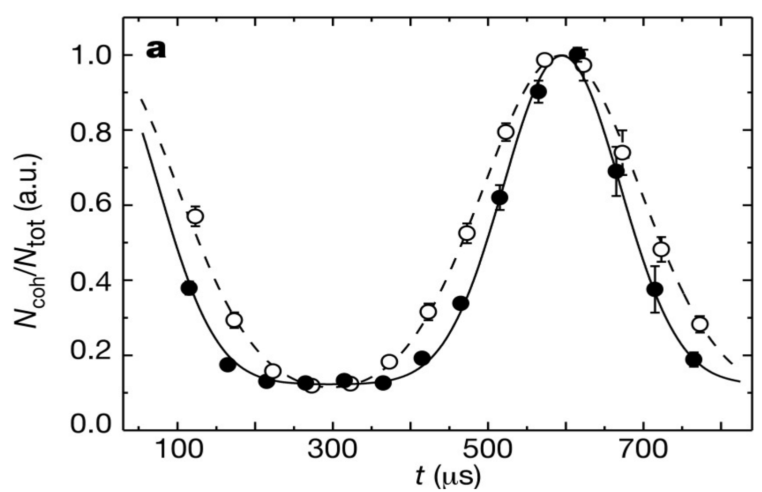
The **Holographic dual** is an **oscillating matter shell** in the gravity theory.

**Entanglement entropy** is a **bridge** between **string theory** (general relativity) and **cond-mat**.




[Lopez-Abajo-Arrastia-da Silva-Mas-Serantes 14]




[Greiner-Mandel-Hänsch-Bloch 02]

# Summary

Entanglement entropy is a **measure for quantum entanglement**.

In **condensed matter** systems entanglement entropy can be used to identify new **quantum phases**. **(quantum order parameter)**

Except in **1+1 dim. CFTs** computing entanglement entropy directly in **QFT's** is **problematic**.

The **AdS/CFT** correspondence provides a **powerful alternative** which works also in **higher dimensions**.

**Entanglement entropy = area of extremal "surfaces"**

**Thermalization** in CFT = **black hole formation** in gravity theory.

Entanglement entropy can be a useful **bridge between general relativity** (string theory) and **condensed matter physics** (and **experiment?**)