1 loop partition function in conformal gravity

I. Lovrekovic

QUANTUM AND GRAVITY
Vienna Central European Seminar 2015

Content

- Introduction, conformal gravity and AdS/CFT
- Conformal gravity
- 1 loop partition function of conformal gravity
- Evaluation of 1 loop partition function via heat kernel
- Conclusion
Introduction:
Conformal Gravity and AdS/CFT

- Earlier theoretical work on CG:
  - **CG** has been studied by J. Maldacena [Einstein gravity from conformal gravity]. One can obtain EG upon imposing suitable boundary conditions on CG.
  - CG emerges from twistor string theory, as a boundary counterterm from 5 dimensional Einstein gravity (EG).
  - Conformal symmetry could according to ’t Hooft give us deeper insight into physics at Planck scale.
  - On theoretical grounds CG has proven to be well defined, however it has to be further studied.
Introduction: Conformal Gravity and AdS/CFT

- Model for gravity at large distances, [Grumiller, 2010]

- Conformal gravity has on the phenomenological grounds proven to describe the galactic rotation curves without addition of dark matter [P. Mannheim, 1989]

**Earlier theoretical work on AdS/CFT:**

- AdS/CFT correspondence was introduced by [J. Maldacena, 1997]. It is shown to work on examples, and extends to other gravity and field theories, e.g. AdS3/LCFT(2) [D. Grumiller and O. Hohm, 2009], Gauge/Gravity duals [Erdmenger at al. 2007]
Introduction: Conformal Gravity and AdS/CFT

- **Holographic principle**: quantities in $d+1$ dimensional AdS space have its duals on the $d$ dimensional boundary of that space

  \[
  \text{AdS} \rightarrow \text{CFT} \\
  \text{bulk} \rightarrow \text{boundary} \\
  \phi_0(x) = \phi(\rho = 0, x) = \phi_{\partial \text{AdS}}(x)
  \]

- **Partition function**, crucial role in AdS/CFT conjecture

  \[
  Z_{QFT}[\phi_0] = Z_{\text{gravity}}[\phi \rightarrow \phi_0]
  \]
Conformal gravity

Conformal transformation: conserves angles, not distances

\[ I_{CG} = \alpha_{CG} \int d^4x \sqrt{|g|} C^\lambda_{\mu\sigma\nu} C^{\mu\sigma\nu}_\chi \] (1)

\[ g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu} \]

\[ C^\lambda_{\mu\sigma\nu} = R^\lambda_{\mu\sigma\nu} - \frac{2}{n-2} \left( g^\lambda_\sigma R_{\nu\mu} - g_{\mu\sigma} R^\lambda_\nu \right) + \frac{2}{(n-1)(n-2)} R g^\lambda_\sigma g_{\nu\mu} \]
1 loop partition function in conformal gravity

\[ \delta^{(1)} S = \alpha \int d^4 x B^{\alpha\beta} \delta g_{\alpha\beta} \]

\[ \delta g_{\mu\nu} = h_{\mu\nu} \quad \delta g^{\mu\nu} = -h^{\mu\nu} \]

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \]

\( B^{\alpha\beta} \) is Bach tensor, EOM of CG

\[ \delta^{(2)} g_{\mu\nu} = 0 \quad \delta^{(2)} g^{\mu\nu} = -\delta h^{\mu\nu} = 2h^{\mu}_{\phantom{\mu} \alpha} h^{\alpha\nu} \]
1 loop partition function in conformal gravity

\[ \delta^{(2)} S = \alpha \int d^4 x \left[ \delta B^{\alpha \beta} \delta g_{\alpha \beta} + B^{\alpha \beta} \delta^{(2)} g_{\alpha \beta} \right] \]

On shell: \[ \delta^{(2)} S = \int d^4 x \delta g_{\alpha \beta} \delta B^{\alpha \beta} \]

Inserting transverse traceless split into \( \delta B^{\alpha \beta} \)

\[ \Rightarrow \delta^{(2)} S = \int d^4 x \left( 8 \Lambda^2 h_{ab}^{TT} h^{TT ab} - 6 \Lambda h^{TT ab} \nabla_c \nabla^c h_{ab}^{TT} \right) \]

\[ + h^{TT ab} \nabla_d \nabla^d \nabla_c \nabla^c h_{ab}^{TT} \)
1 loop partition function of conformal gravity

Aim is to compute partition function of CG:

\[ Z_{CG} = \int Dh_{\mu\nu} \times \text{ghost} \times \text{Exp}(-\delta^{(2)} S) \]  \hspace{1cm} (3)

- explicit separation of gauge modes

- gauge invariance \( \xrightarrow{\text{\#}} \xi \), conformal invariance \( \xrightarrow{\text{\#}} \h \)

- the functional integral over these gauge dof-s yields the volume of the gauge group and has to be eliminated, with which we divide
1 loop partition function of conformal gravity

**Ghost determinant** - given by Jacobians that corresponds to changes of variables:

\[ Dh_{\mu\nu} = Z_{gh} Dh_{\mu\nu}^{TT} D\xi_\mu Dh. \]

\[ D\xi_\mu = J_0 D\xi_\mu^T Ds \]

\[ Dh_{\mu\nu} = JDh_{\mu\nu}^{TT} D\xi_\mu^T DuDs \]

+ conventions for path integral measures, scalar products

\[ Z_{CG} = Z_{gh} \int Dh_{\mu\nu}^{TT} \text{Exp}(-\delta^{(2)} S) \]

\[ Z_{CG} = \frac{[\det(4\Lambda + \nabla^2)]^{1/2} [\det(-3\Lambda - \nabla^2)]^{1/2}}{[\det(-2\Lambda + \nabla^2)]^{1/2} [\det(-4\Lambda + \nabla^2)]^{1/2}} \]
Evaluation of 1 loop partition function via heat kernel

- Partition function consists of scalar mode, transverse vector mode and two symmetric transverse traceless modes (STT).
- We compute them using the method described in [R. Gopakumar, R. K. Gupta, S. Lal, 2011]
- first, recall the relation of one-loop partition function with traced heat kernel:

\[
\ln Z^{(S)} = \ln \det(-\Delta^{(S)}) = - \int_{0}^{\infty} \frac{dt}{t} Tr e^{t\Delta^{(S)}}
\]

\[K^{(S)}(t) \equiv Tr e^{t\Delta^{(S)}}\]
Evaluation of 1 loop partition function via heat kernel

• We want to evaluate the heat kernel on thermal AdS in 4D.

\[ K^{(S)}(\beta, t) = \frac{\beta}{2^3 \sqrt{\pi} t} \sum_{k \in \mathbb{Z}_+} \chi_{(s)}^{SO(3)} \frac{1}{\sinh^3 \frac{k\beta}{2}} e^{-\frac{k^2 \beta^2}{4t} - t(\rho^2 + s)} \]
Evaluation of 1 loop partition function via heat kernel

\[- \log \det \left( -\Delta_{(S)} + m^2_S \right) = \int_0^\infty \frac{dt}{t} K(S) (\beta, t) e^{-m^2_S t} \]

\[ q = e^{-\beta} \]

\[ \log Z_{CG} = \sum_k \frac{-1}{k(1 - q^k)^3} q^{2k\beta} \left( 4q^{2k} - 5 - 5q^k \right) \]

- Each of the exponentials belongs to a determinant which can be identified.
- Partition function on \( S^1 \times S^3 \)
Conclusion

• We obtained the one loop partition function of conformal gravity

• it contains part that can be recognised as Einstein gravity partition function and contribution that can be recognised as it originates from the conformal ghost

• we compared it with the partition function on $S^1 \times S^3$
Thank you!
For compact Lie groups $G$ and $H$, with $H$ subgroup of $G$, the coset space $G/H$ is constructed through right action of $H$ on elements of $G$: $G/H = \{gH\}$.

- Section in $G$ is $\sigma : G/H \rightarrow G$
- $\sigma(gH) = g_0$ is element of $gH$
- Heat kernel on $\Gamma \backslash G/H$
- $\gamma \in \Gamma$ acts on points $x = gH \in G/H$ by $\gamma : gH \rightarrow \gamma \cdot gH$
  $\Rightarrow \sigma(\gamma(x)) = \gamma \cdot \sigma(x)$